

MAT123

Symmetry

Even/Odd Functions

A function is even if $f(-x) = f(x)$ for all x -values in the domain

$$\begin{aligned}\text{ex : } g(x) &= x^4 - 3x^2 \\ g(-x) &= (-x)^4 - 3(-x)^2 \\ &= x^4 - 3x^2 = g(x)\end{aligned}$$

since $g(-x) = g(x)$
 $g(x)$ is an even function

A function is odd if $f(-x) = -f(x)$ for all x -values in the domain

$$\begin{aligned}\text{ex : } h(x) &= x^3 - 6x \\ h(-x) &= (-x)^3 - 6(-x) \\ &= -x^3 + 6x \quad \text{factor out negative} \\ &= -(x^3 - 6x) \\ &= -h(x)\end{aligned}$$

$$\begin{aligned}(-x)^3 &= (-x)(-x)(-x) \\ &= -x^3\end{aligned}$$

since $h(-x) = -h(x)$
 $h(x)$ is an odd function

Example of neither:

$$f(x) = x^2 + 2x + 1$$

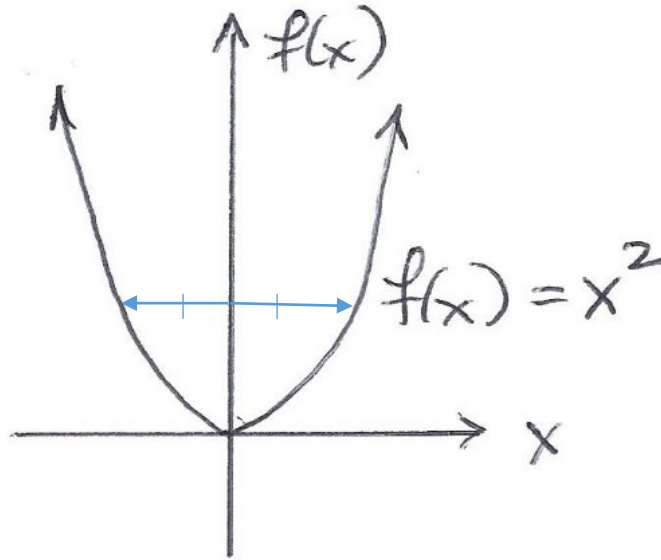
$$f(-x) = x^2 - 2x + 1 \neq f(x) \neq -f(x)$$

$f(x)$ is neither even nor odd

Even/Odd - Symmetry on a Graph

Even functions have y-axis symmetry:

ex. $f(x) = x^2$



Odd functions have origin symmetry:

ex. $f(x) = x^3$

